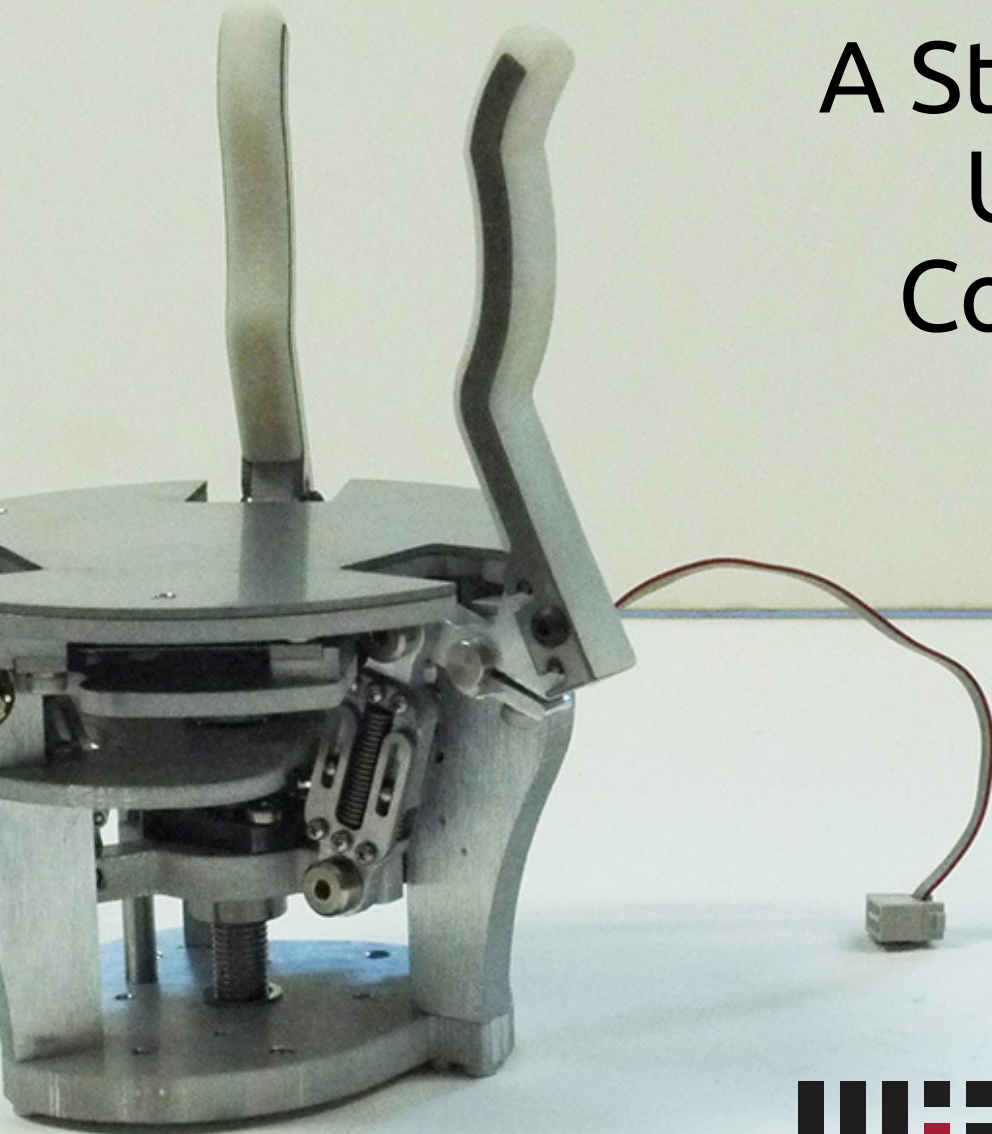


# Pusher-Slider: A Story of Hybrid and Underactuated Contact Dynamics

October 14, 2016  
IROS – Workshop  
Daejeon

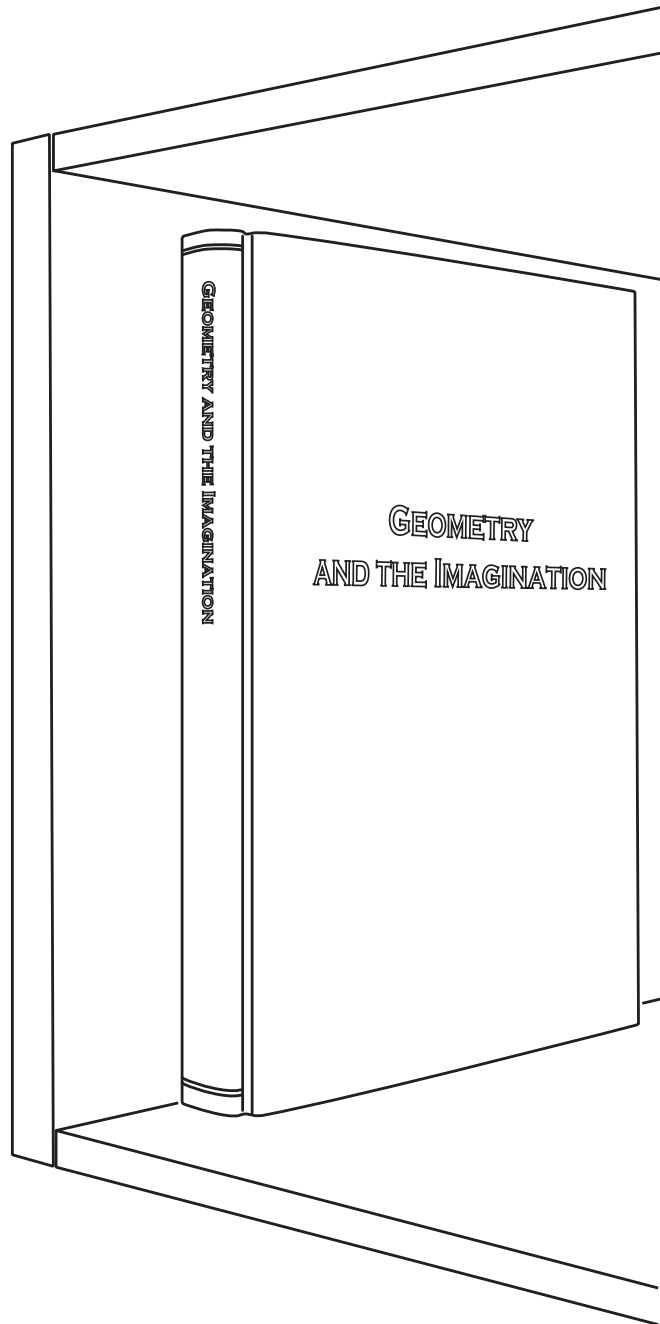
Alberto Rodriguez



Massachusetts  
Institute of  
Technology

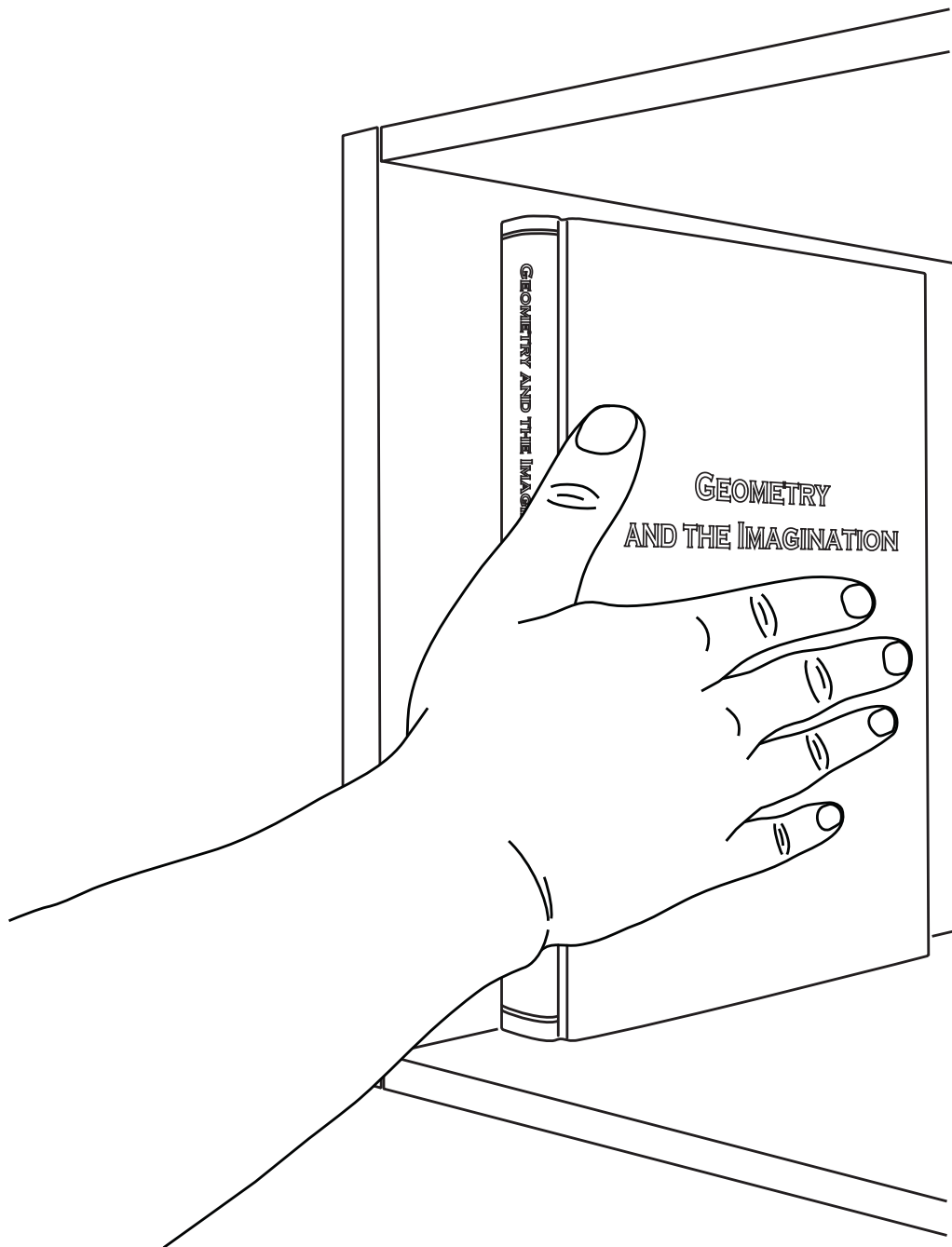


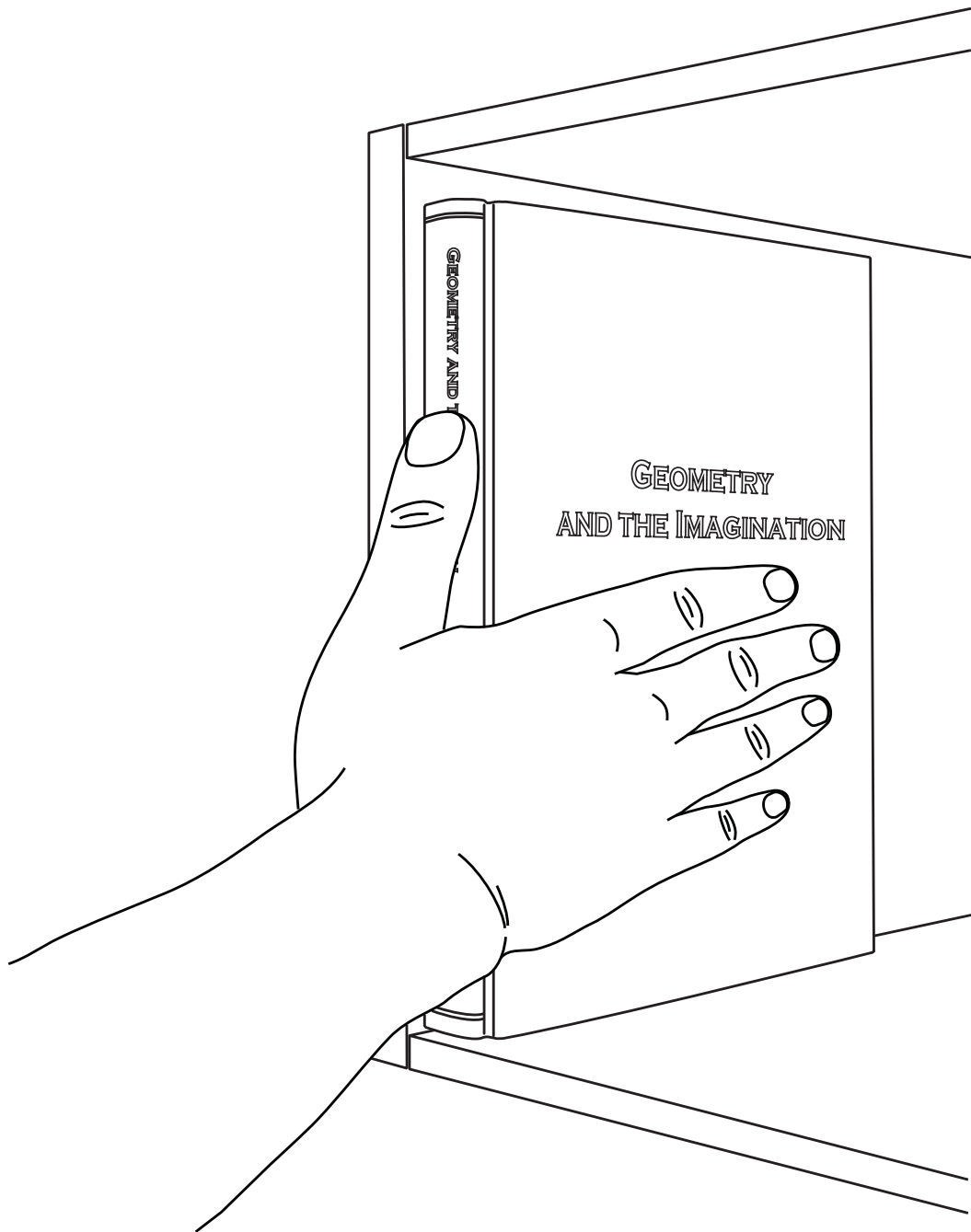
MCube  
Lab



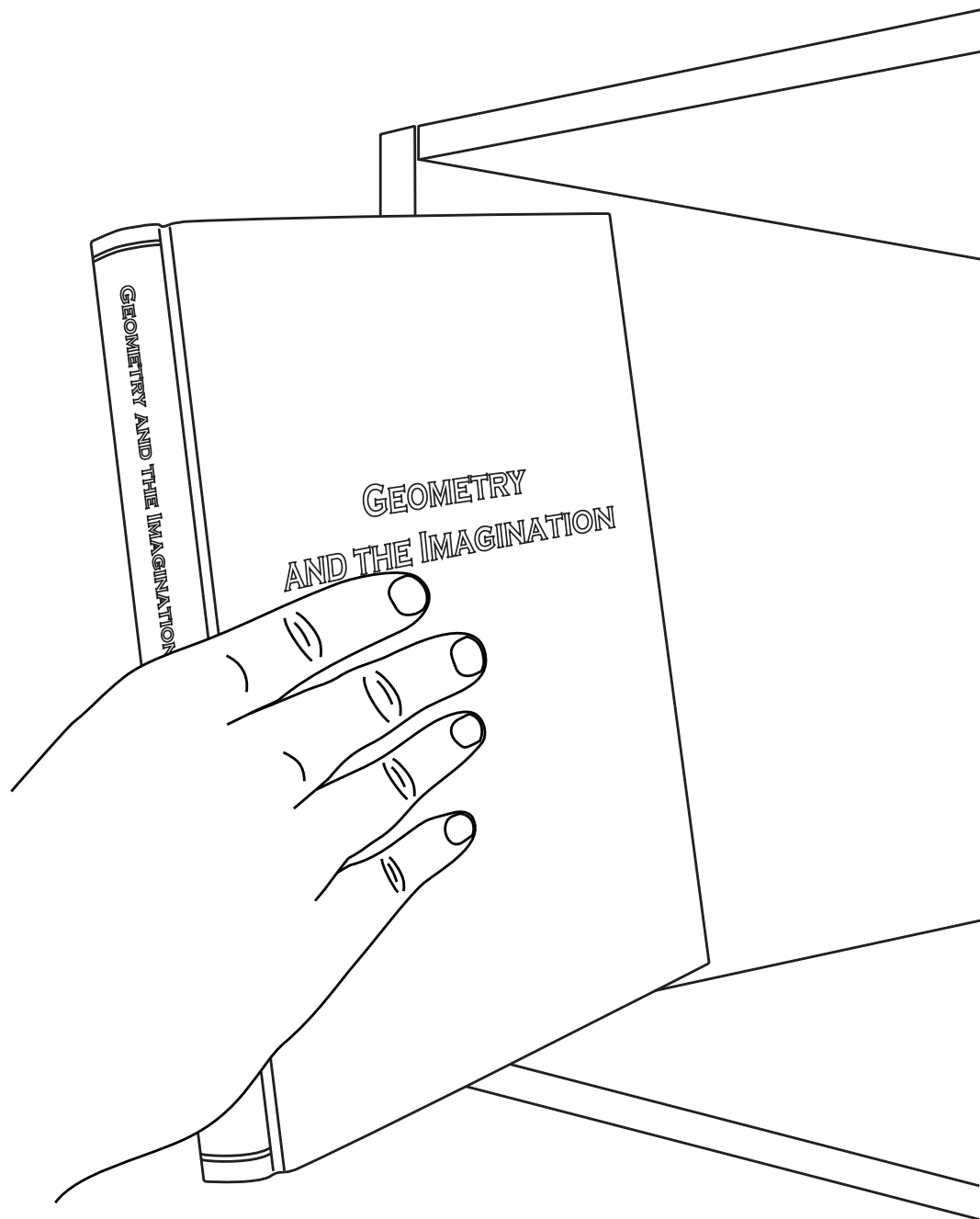
GEOMETRY AND THE IMAGINATION

GEOMETRY  
AND THE IMAGINATION











# Challenges

## Underactuation

Control over a limited set of forces.

*“To pull the book, we have to push on it”*

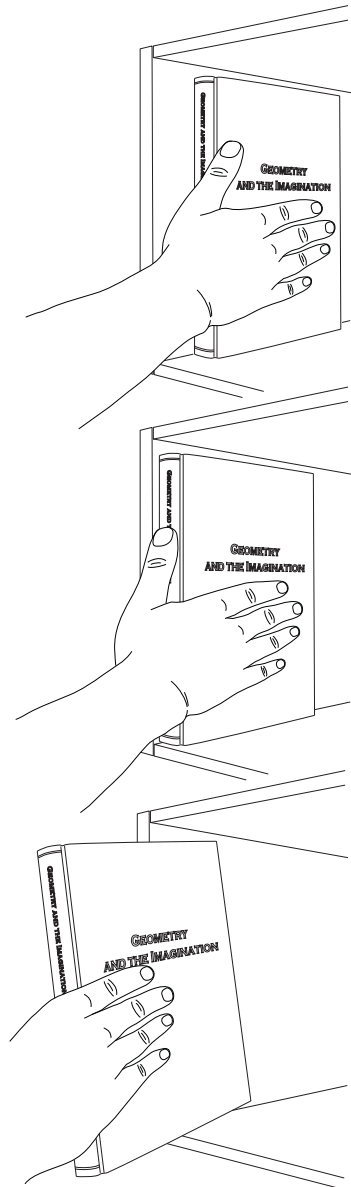
➤ Horizon

## Hybridness of contact

Different contact modalities (stick, slip, separation).

*“The hand contacts, then sticks, then slides, then sticks, ... , then grasps”*

➤ Combinatorial



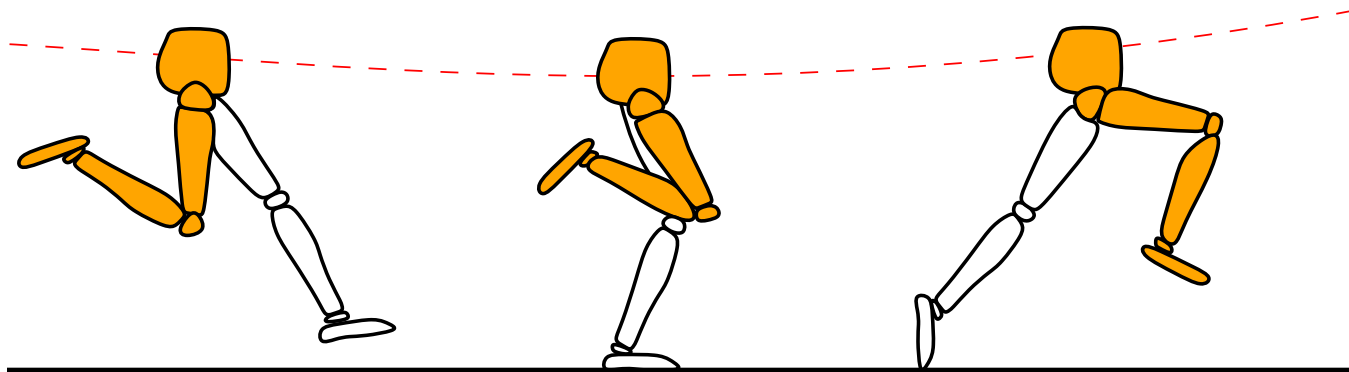


# Challenges

## *Underactuation and Hybridness*

### **Locomotion**

Everywhere, all the time.



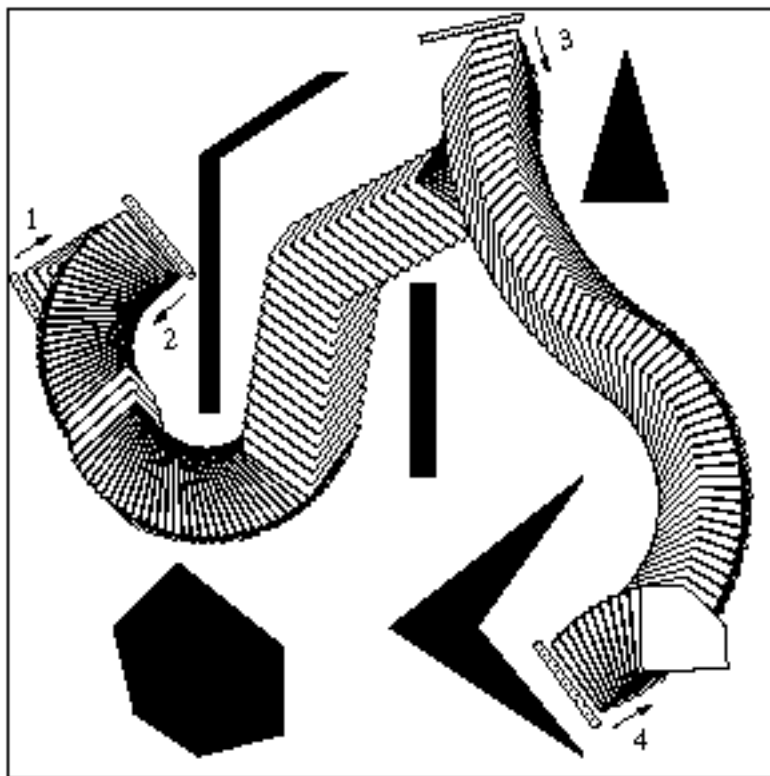


# Challenges

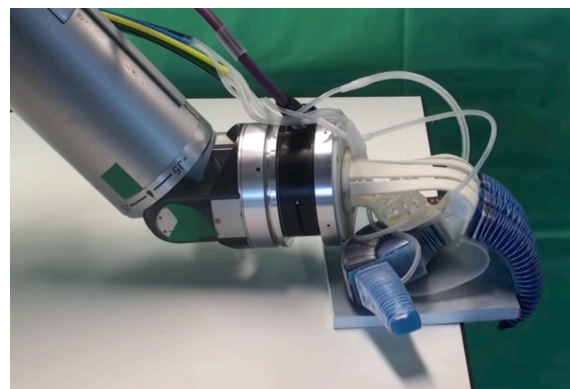
## *Underactuation and Hybridness*

### Manipulation

Usually looking for open loop (i.e. implicit feedback) stable solutions.



*[Lynch and Mason, 1996]*



*[Brock's gang, 2016]*



# Challenges

*Underactuation and Hybridness*

## Manipulation

What about closed-loop (i.e. explicit feedback)?







# Challenges

*Underactuation and Hybridness*

## Manipulation

What about closed-loop (i.e. explicit feedback)?





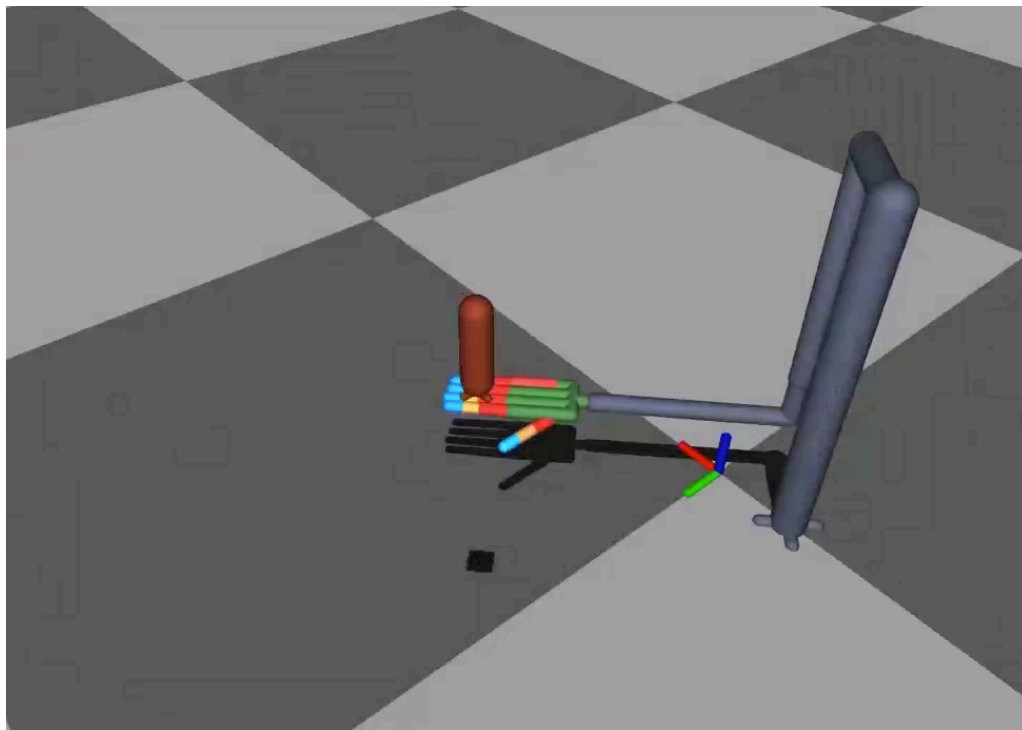
# Challenges

## *Underactuation and Hybridness*

### Manipulation

Explicitly considered in trajectory optimization through contact.

- ✓ Either off-line or on-line with smoothed-out models.
- ✓ Usually inspired by locomotion solutions.



*[Kumar et al., 2014]*





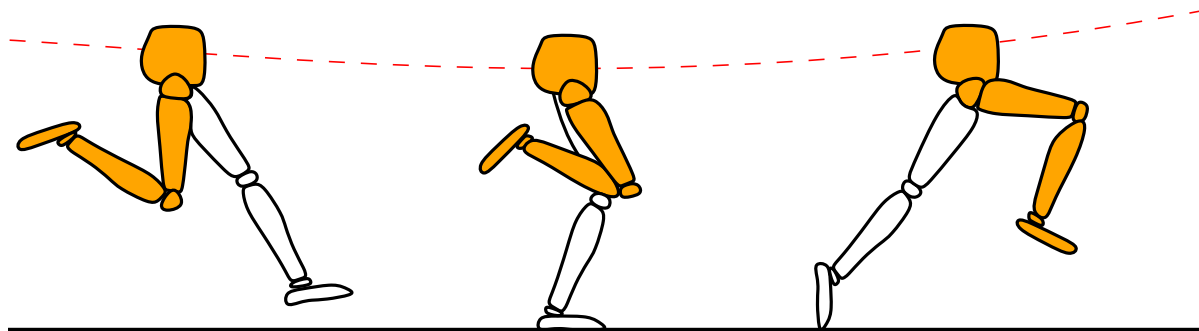
# Challenges

## *Underactuation and Hybridness*

### Manipulation

What constitutes a good feedback controller for manipulation?

- ✓ Addresses **underactuation** and **hybridness**.
- ✓ Allows for **sliding** at contact.
- ✓ Cares less about **temporal** resolution.

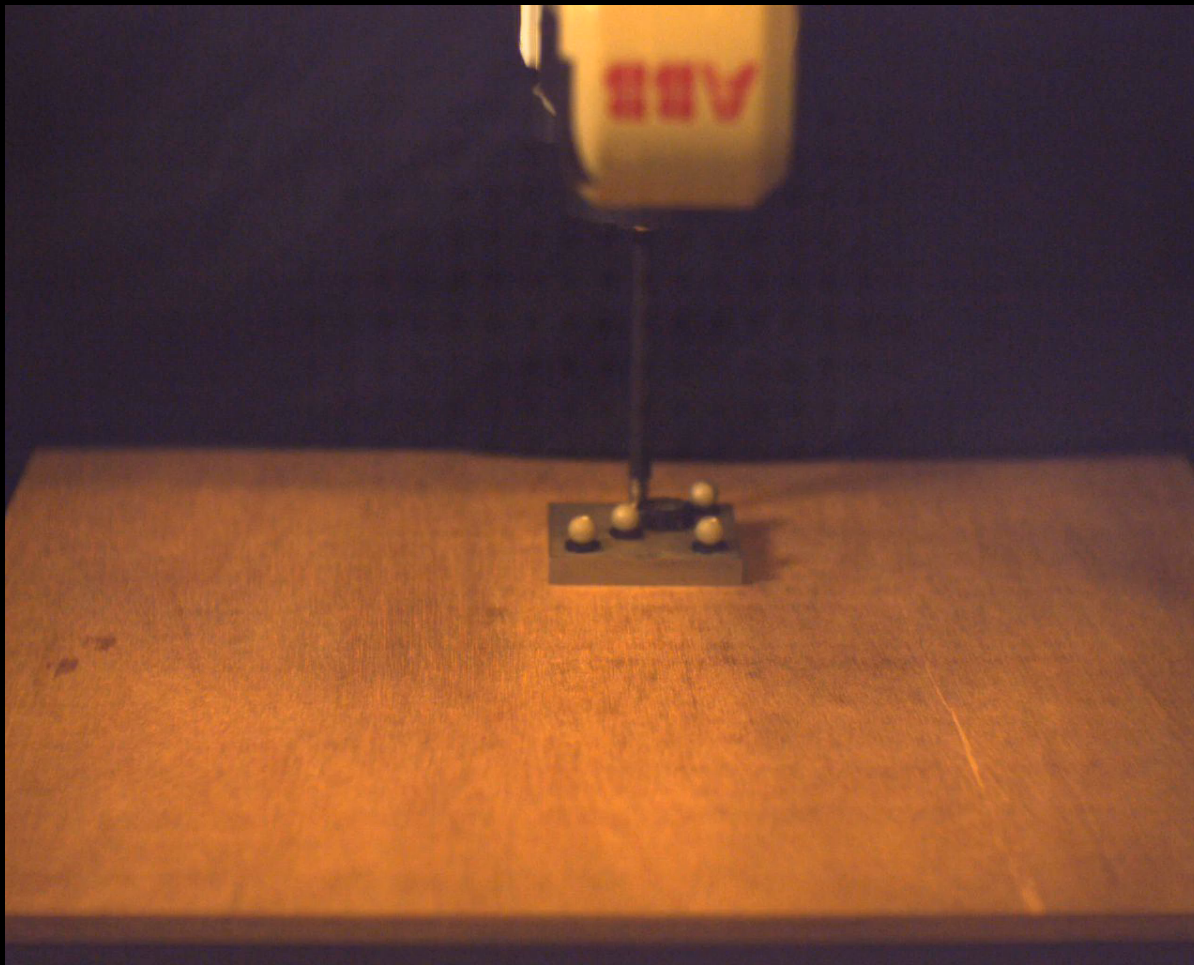




# Pusher-Slider System

*Simple manipulation task*

Control the motion of a sliding object using a single friction contact point





# Pusher-Slider System

## “Simple” Manipulation Problem

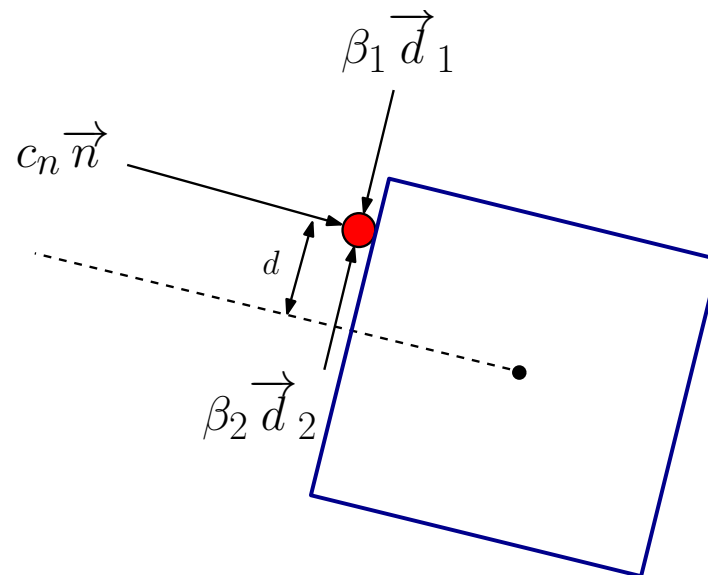
Incorporates:

### Underactuation

- Only push, no pull.
- Force limited to friction cone.

### Hybrid Dynamics

- Modes: Stick, Slide, Separate.
- Available forces depend on mode.



- |           |   |                                       |
|-----------|---|---------------------------------------|
| $c_n$     | = | Normal force along $\vec{n}$          |
| $\beta_1$ | = | Frictional force along $\vec{d}_1$    |
| $\beta_2$ | = | Frictional force along $\vec{d}_2$    |
| $d$       | = | Relative position of pusher to slider |

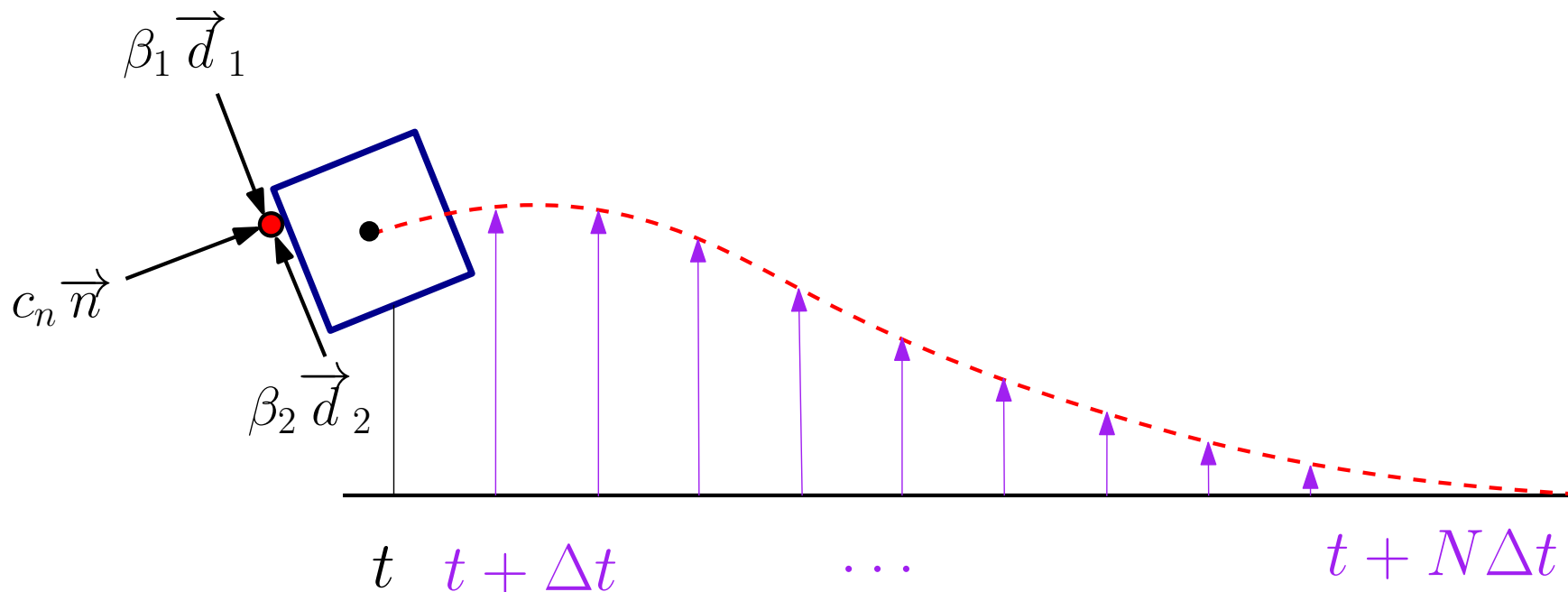


# Pusher-Slider System

## *Problem Formulation*

### Example: Line following

- Model Predictive Control approach.
- Minimize error over finite horizon.
- Enforce constraints on control inputs.



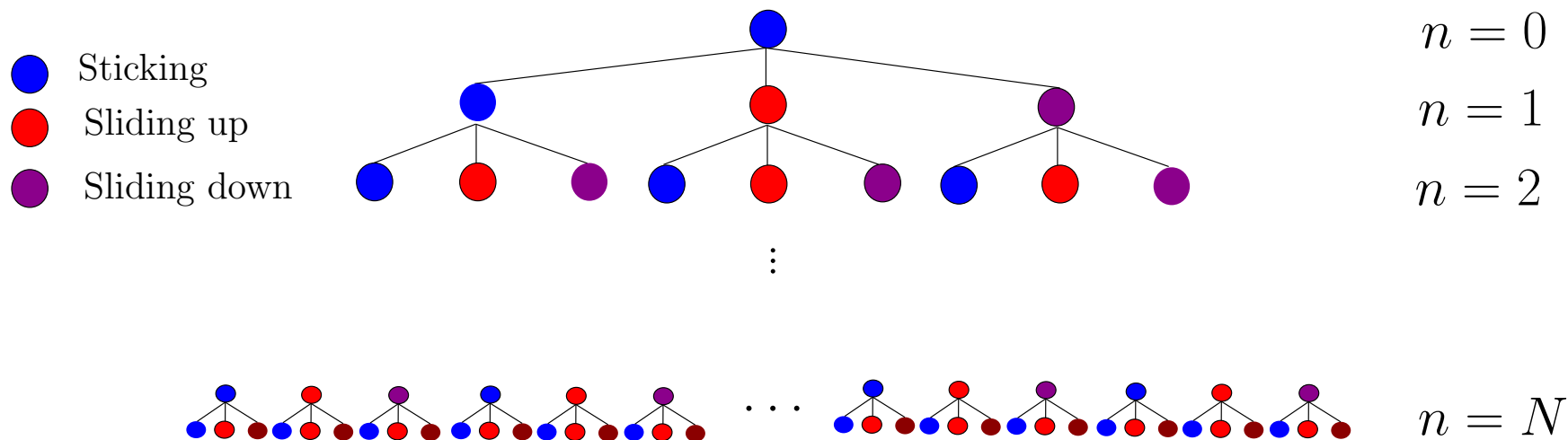


# Pusher-Slider System

## *Problem Formulation*

### Possible Solution 1

Full contact mode enumeration (naïve approach)



- ✓ Tree of optimization programs.
- ✓ Scales exponentially due to contact hybridness.

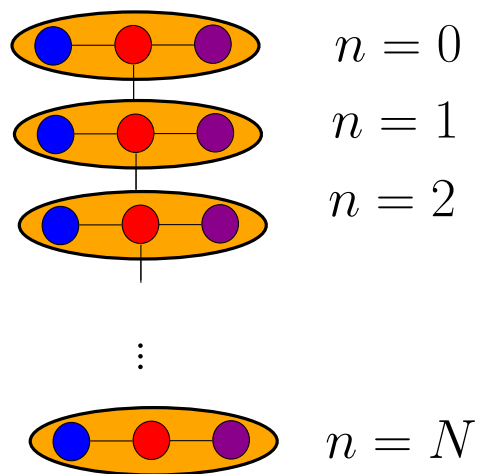


# Pusher-Slider System

## *Problem Formulation*

### Possible Solution 2

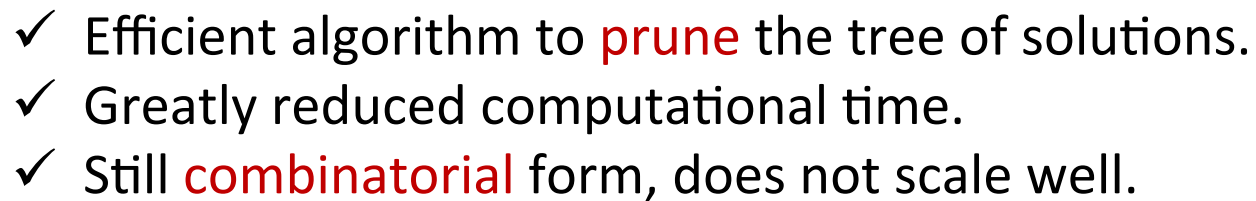
Complementarity formulations of contact



- ✓ Standard in simulation and trajectory planning.
- ✓ Avoids combinatorial nature.
- ✓ Non-convex constraints, and time consuming.



# Mixed-Integer Programming





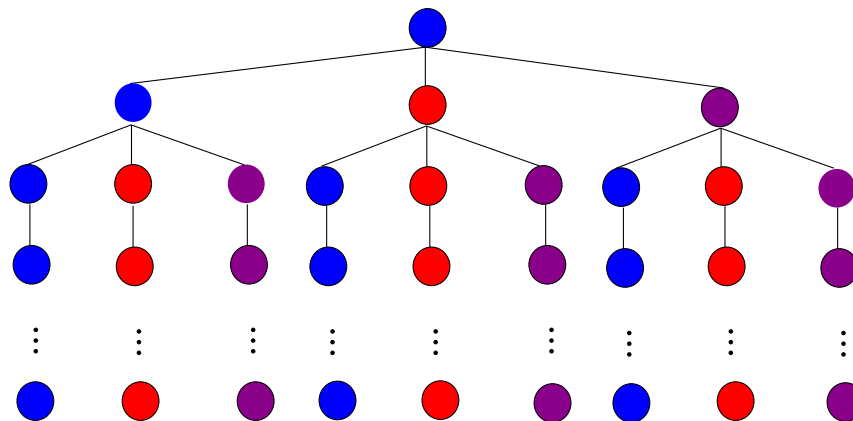


# Pusher-Slider System

## *Problem Formulation*

### Possible Solution 4

Family of Modes



$$3^N \rightarrow 9$$

- ✓ Predefine **primitive hybrid behaviors** (each like a smooth controller).
- ✓ Eliminates combinatorial form.
- ✓ Only **approximate**.
- ✓ Need to determine **off-line** primitive behaviors.

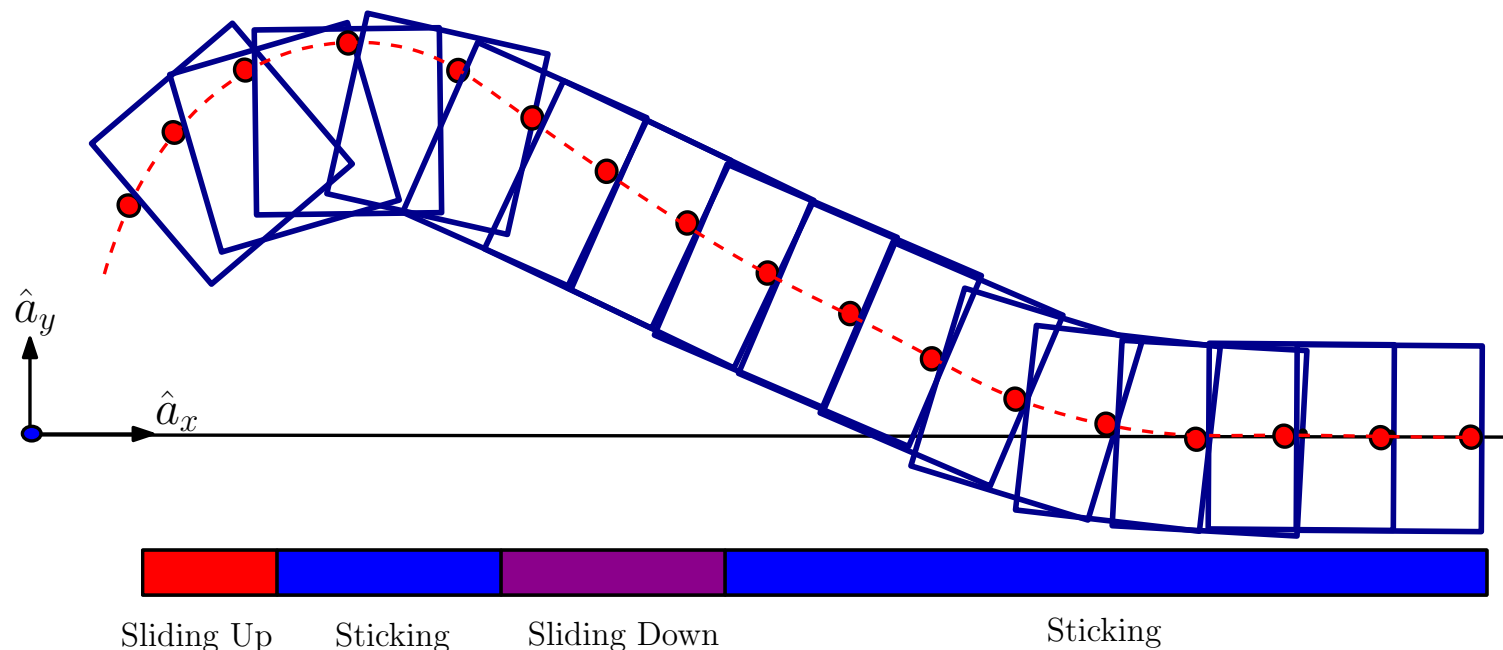


# Pusher-Slider System

## *Problem Formulation*

### Possible Solution 4

Example of a possible primitive mode in the family



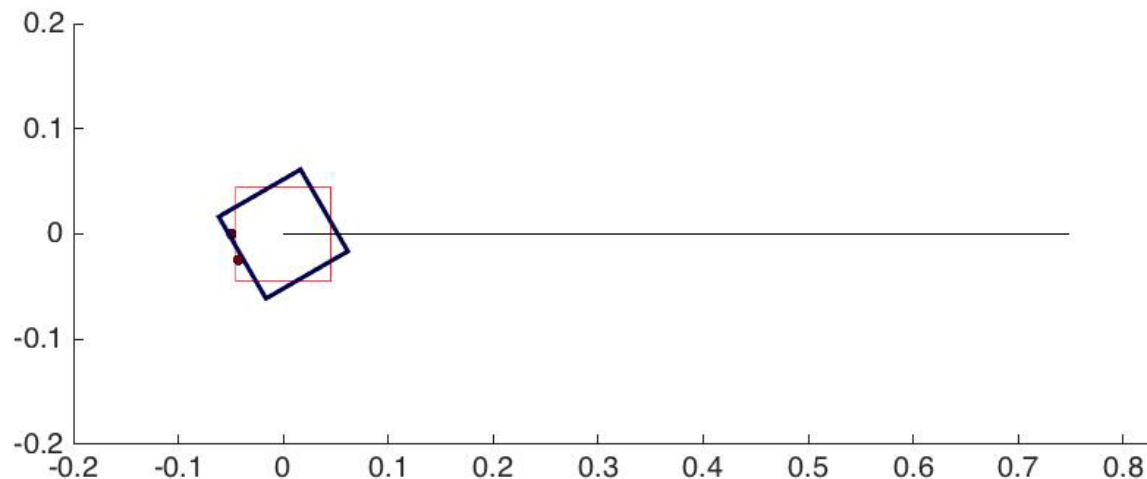
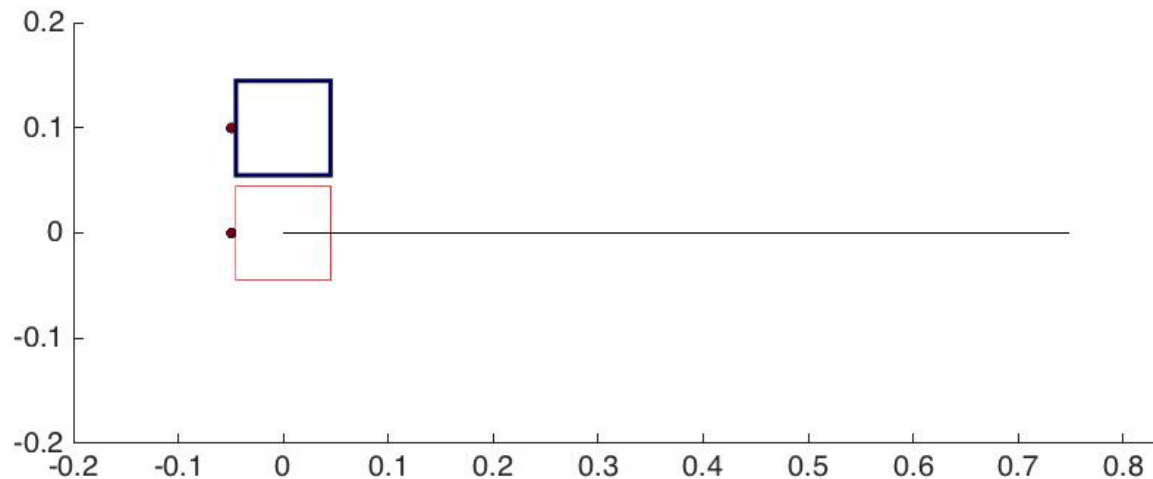
- ✓ We fix the **sequence** of mode transitions.
- ✓ Optimization determines **controls** during each of those.



# Pusher-Slider System

## Example: Follow a line

Family of Modes in action (9 modes – 50Hz)

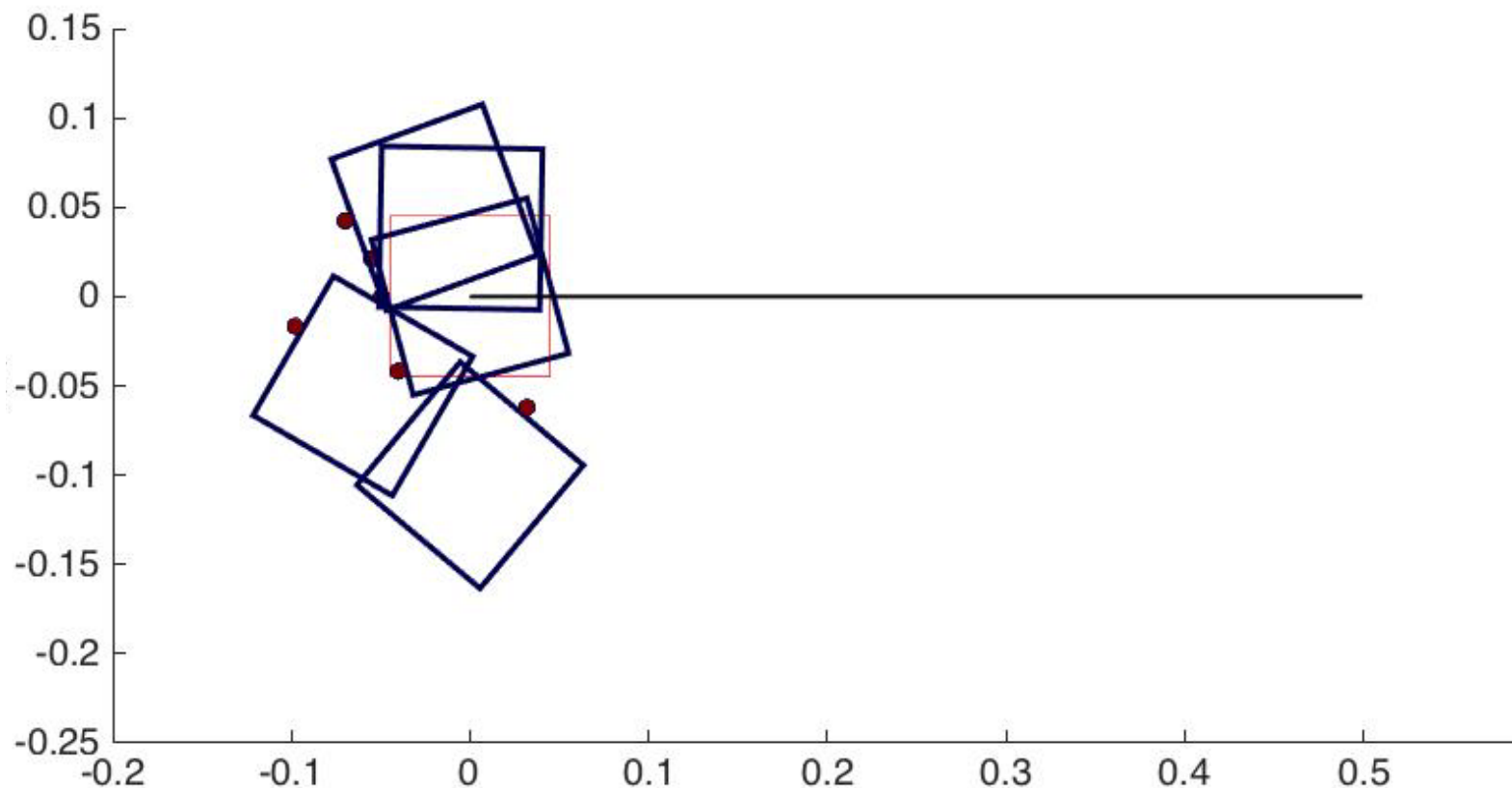




# Pusher-Slider System

## Example: Follow a line

Family of Modes in action (9 modes – 50Hz)

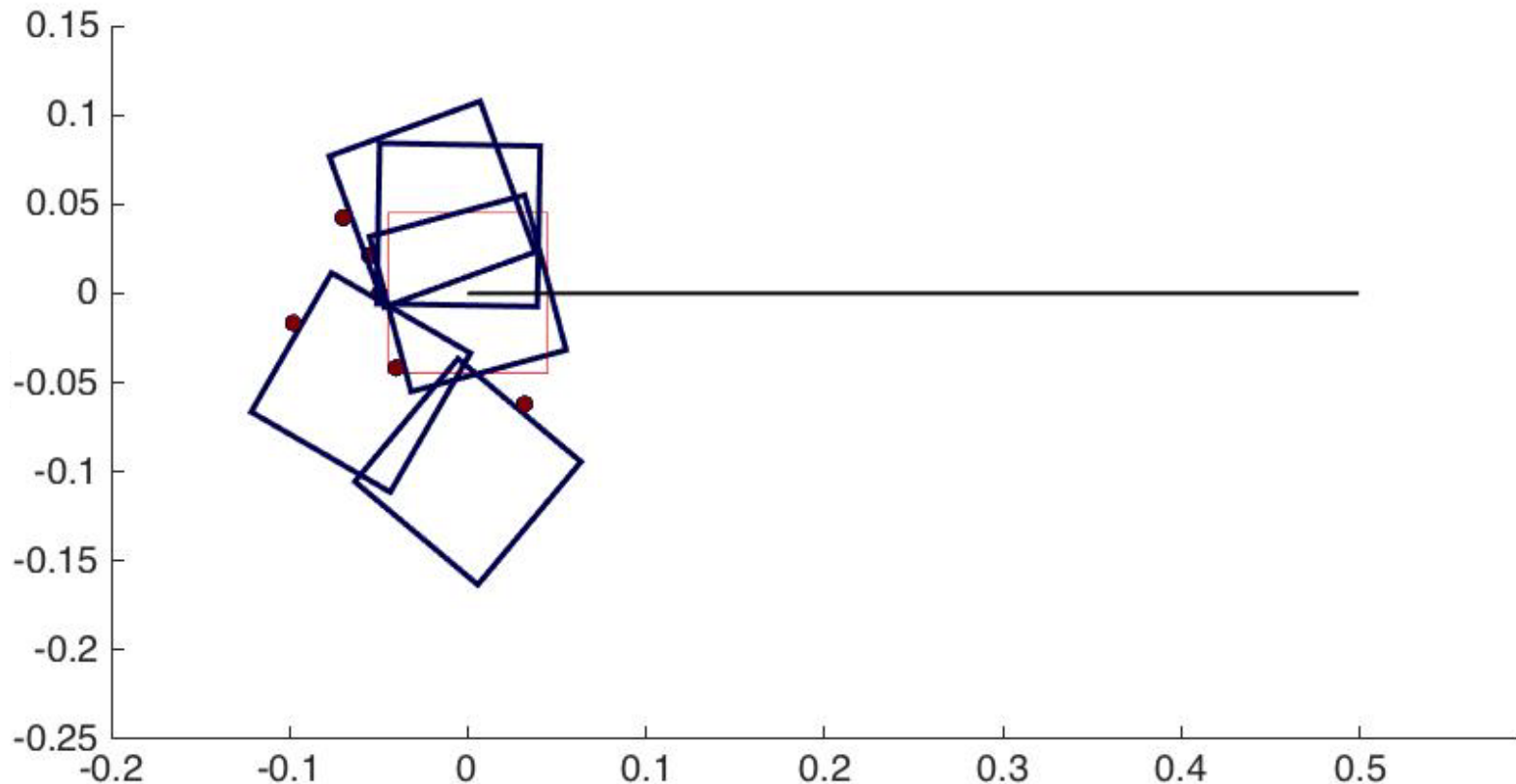




# Pusher-Slider System

## Example: Follow a line

Family of Modes in action (9 modes – 50Hz – 30% noise in coeff. friction)

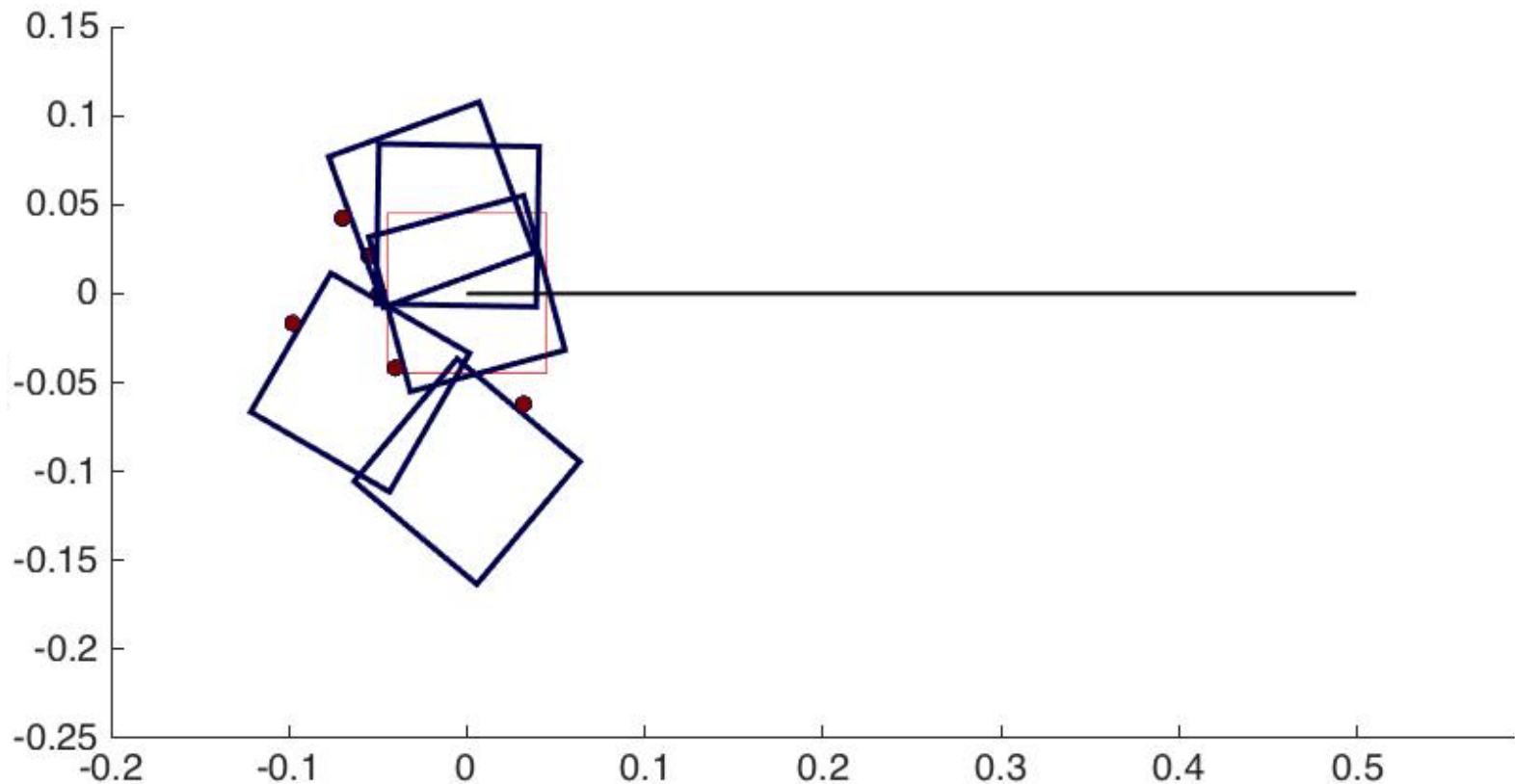




# Pusher-Slider System

## Example: Follow a line

Family of Modes in action (9 modes – 50Hz – 5mm noise in observations)







# Real Experiments

*Does it work?*

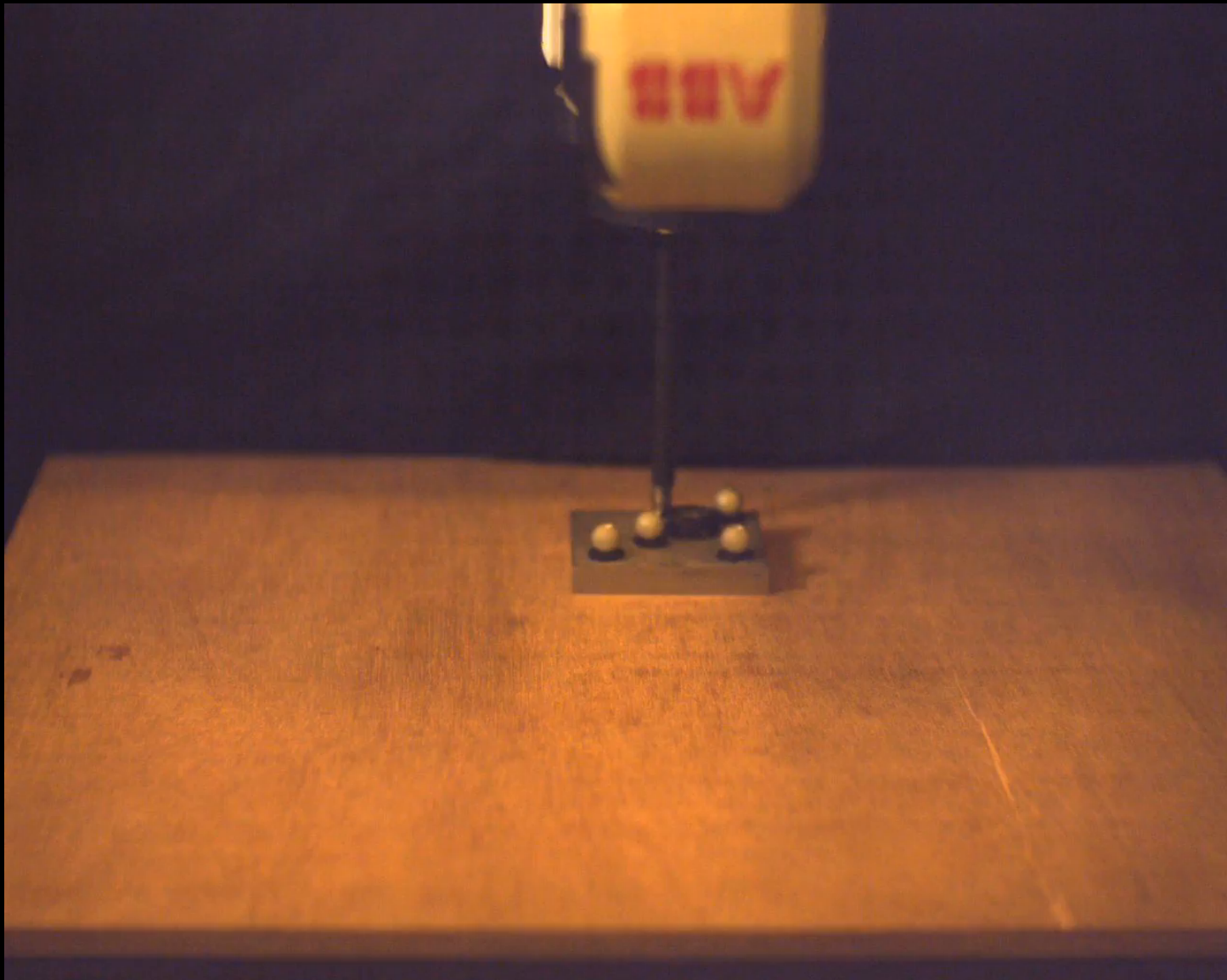






# Real Experiments

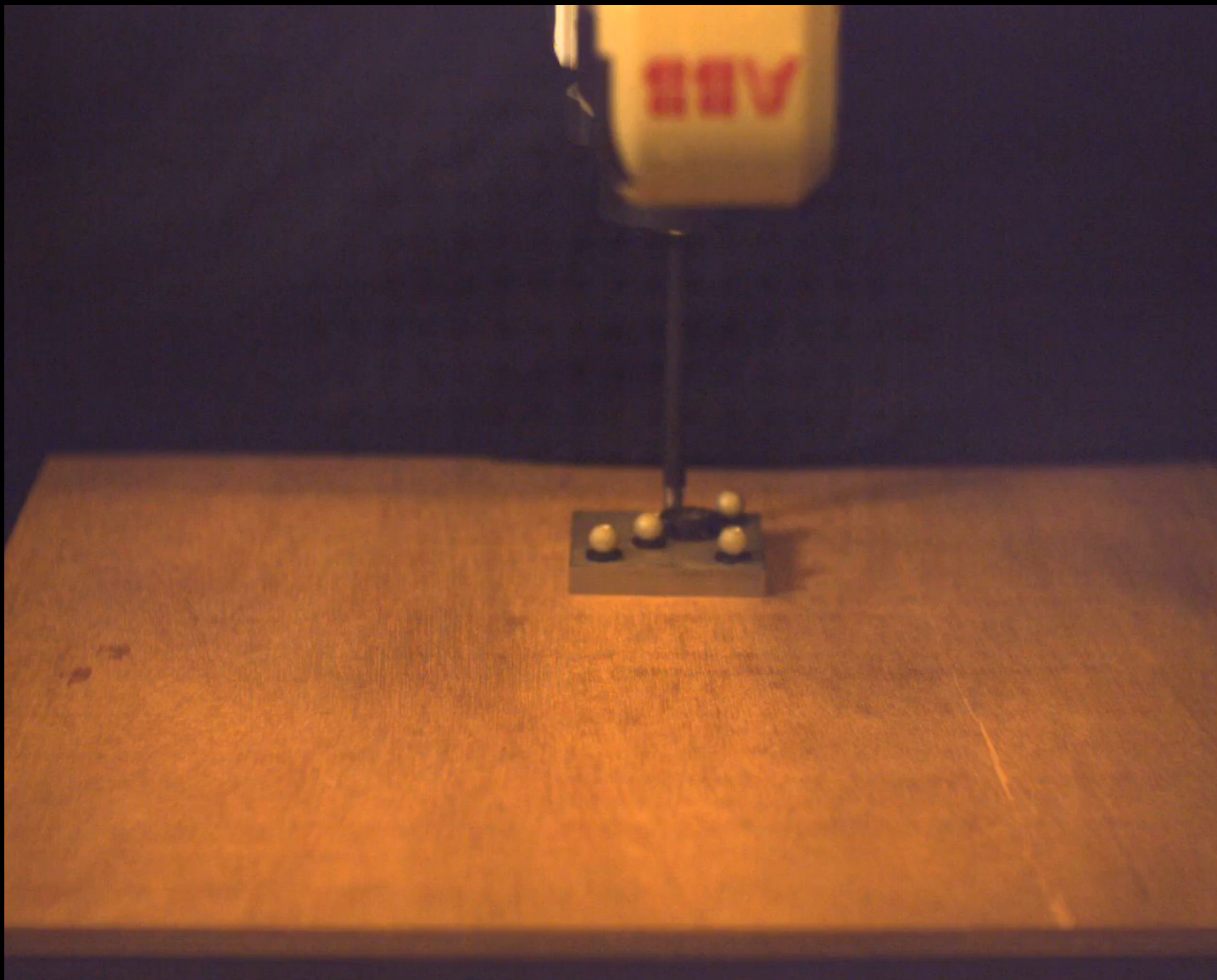
*Line following with feedback*





# Real Experiments

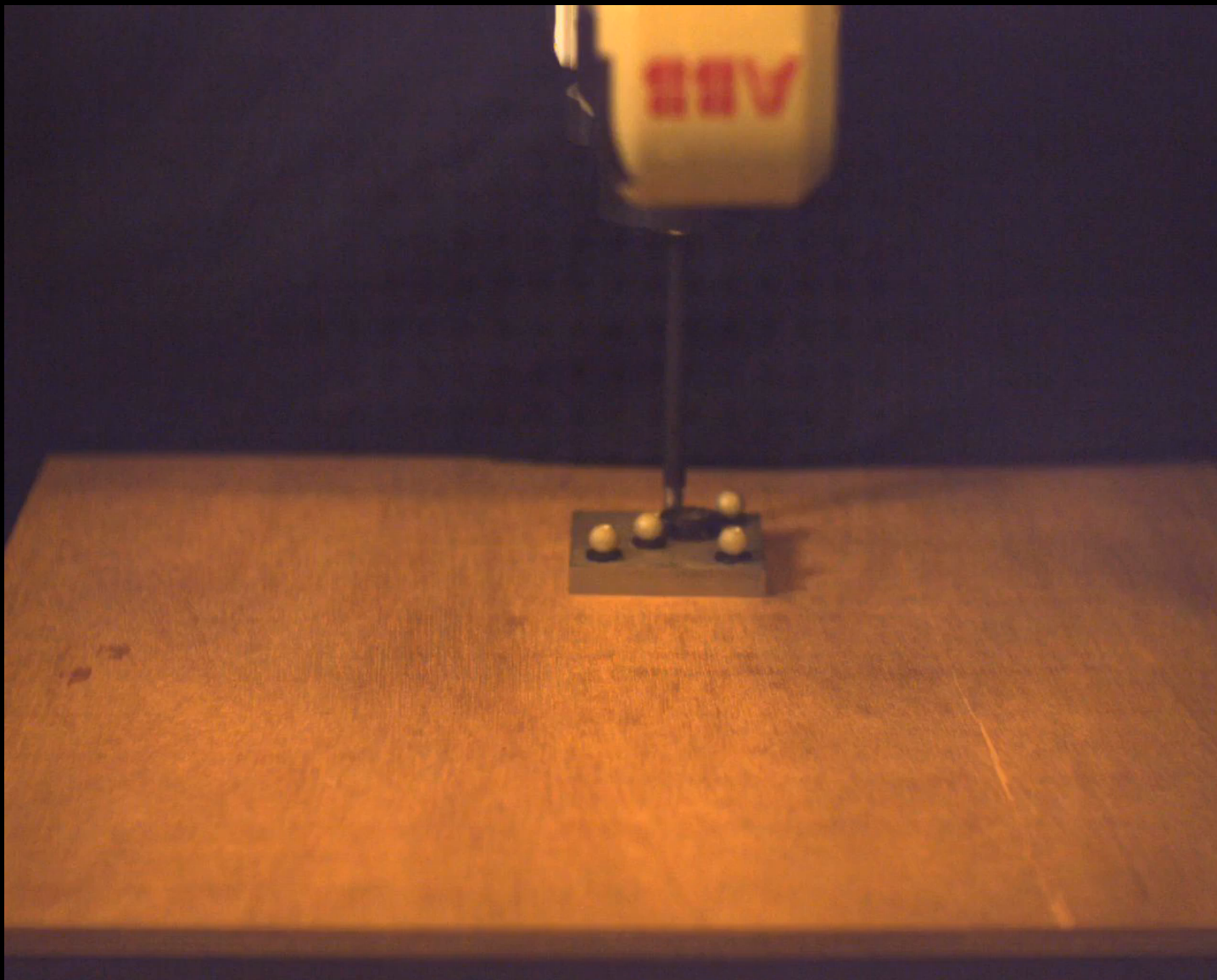
*External Perturbations*





# Real Experiments

*External Perturbations  $1/8 X$*

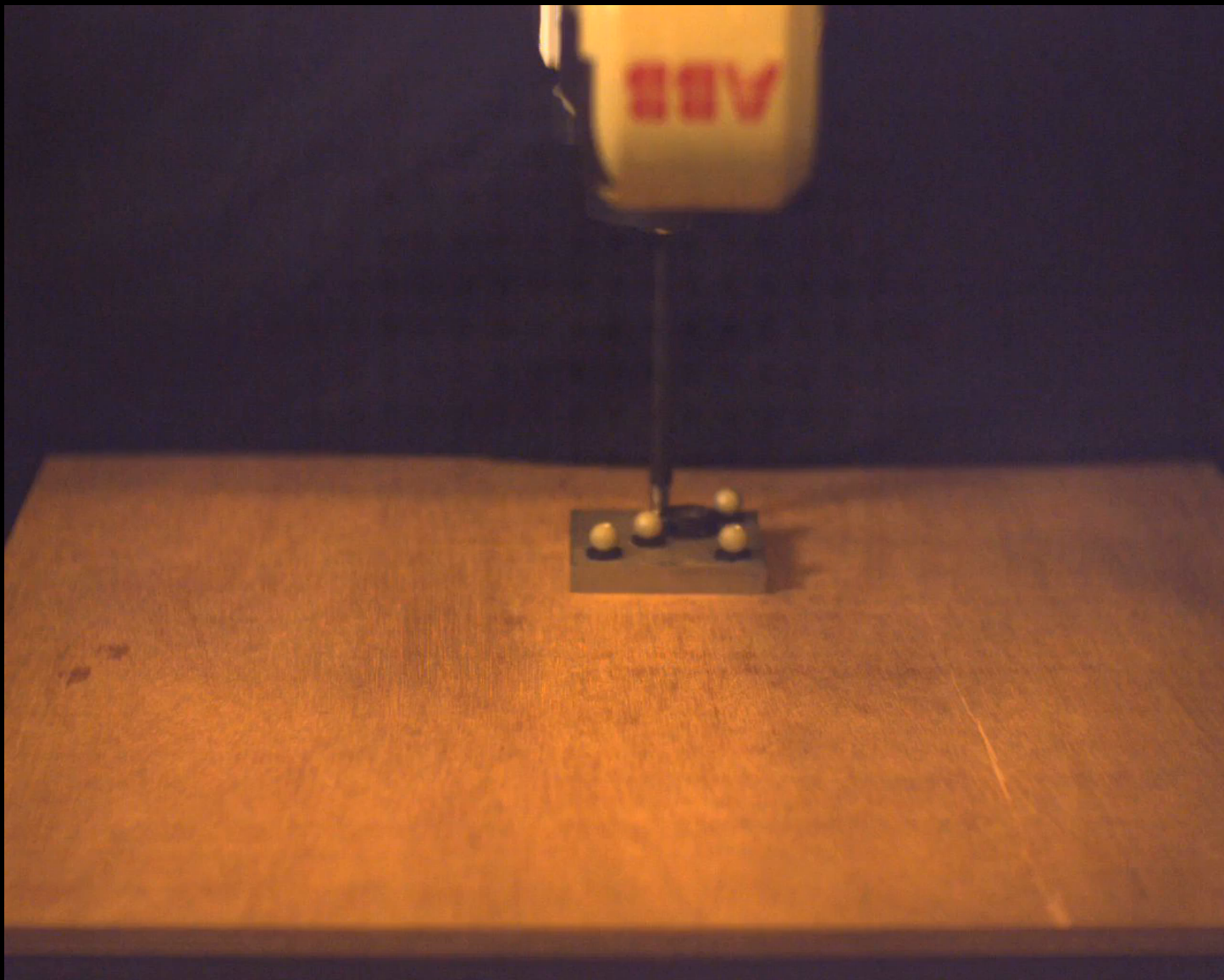






# Real Experiments

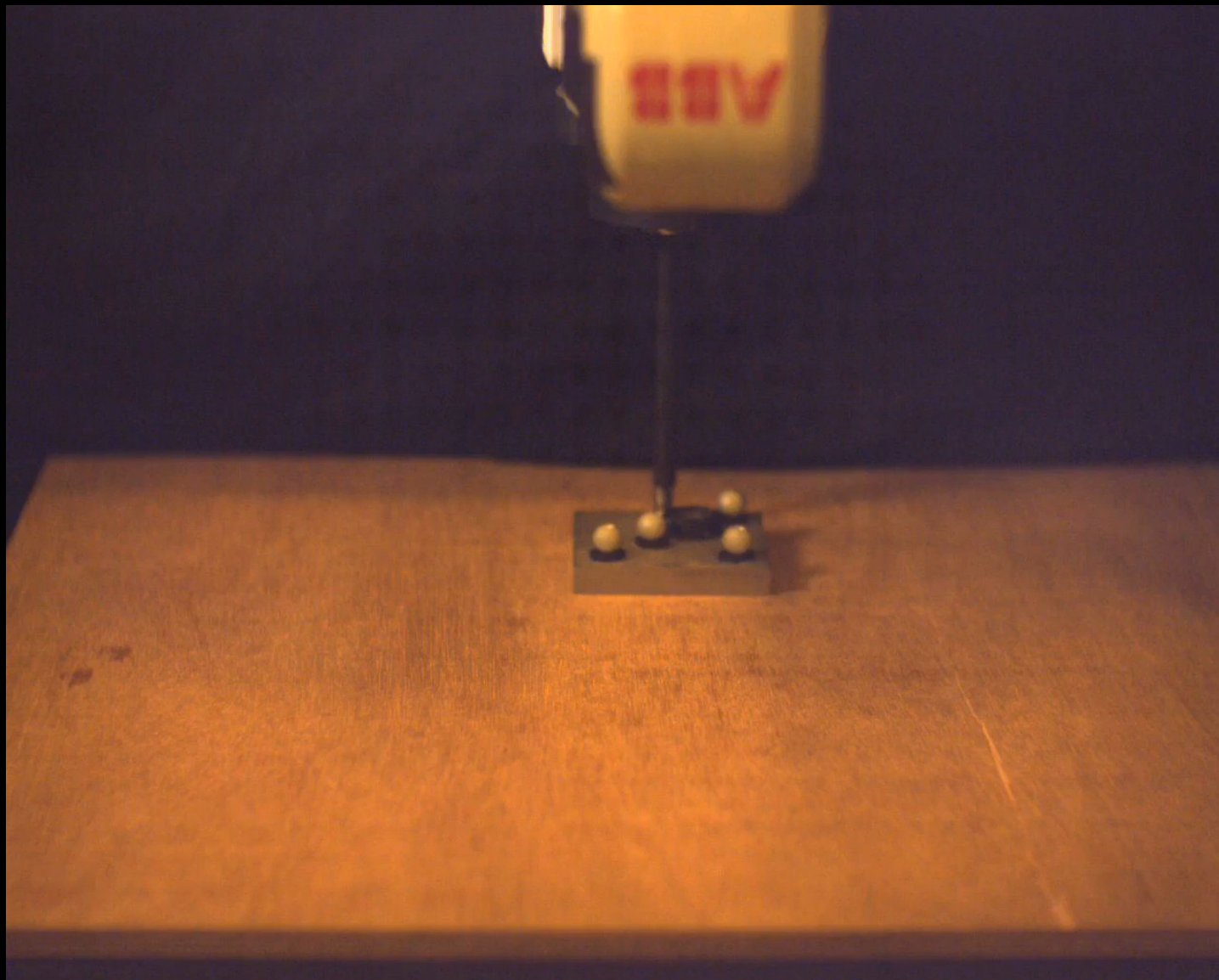
## *External Perturbations*





# Real Experiments

*External Perturbations  $1/8 X$*





# Pusher-Slider System

## Example: Follow a target

Family of Modes in action (3 modes – 50Hz)

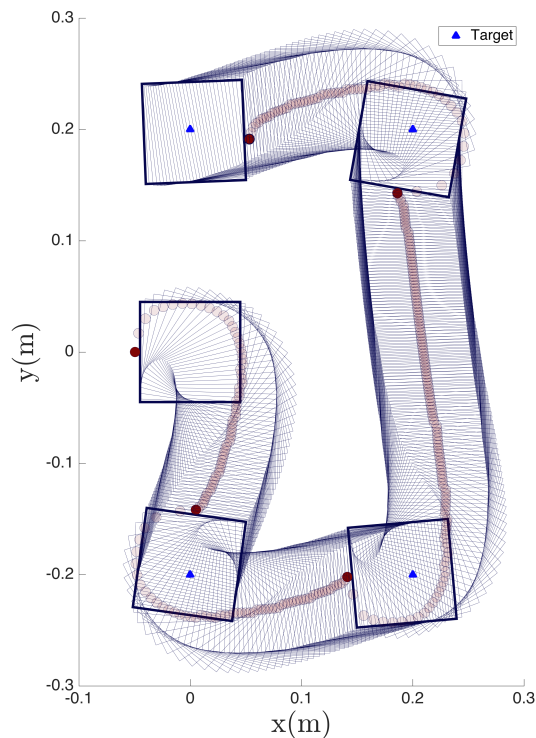




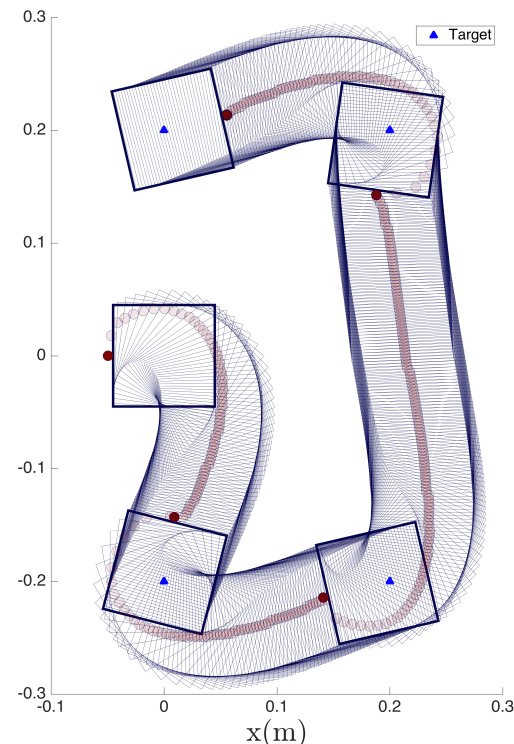
# Pusher-Slider System

## Example: Follow a target

Family of Modes vs. Mixed-Integer Programming



Mixed-Integer Programming (0.7s)



Family of Modes (0.02s)

Almost identical behavior → Key for **Manipulation**.





# Real Experiments

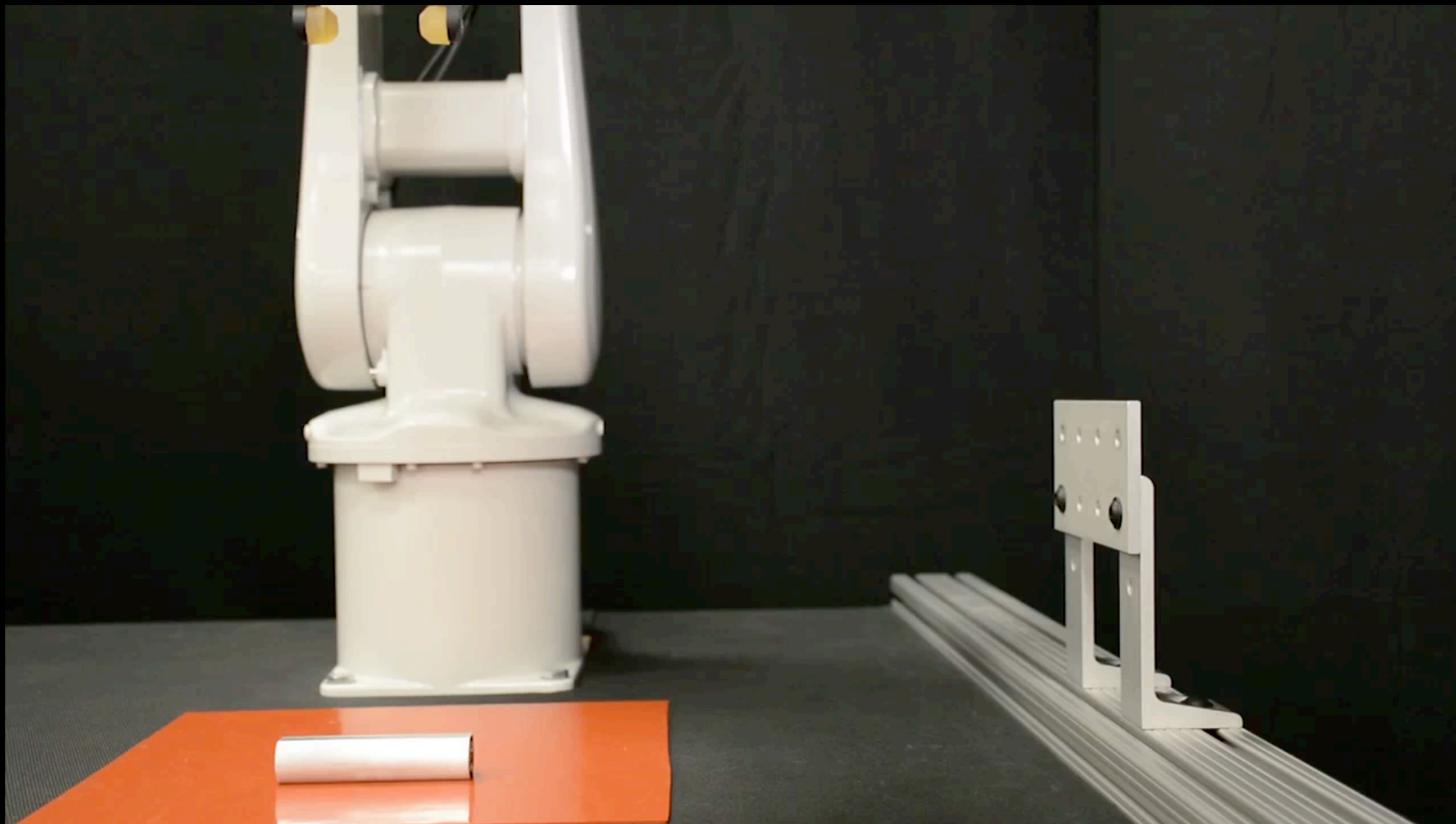
*Point tracking*





# Future Work

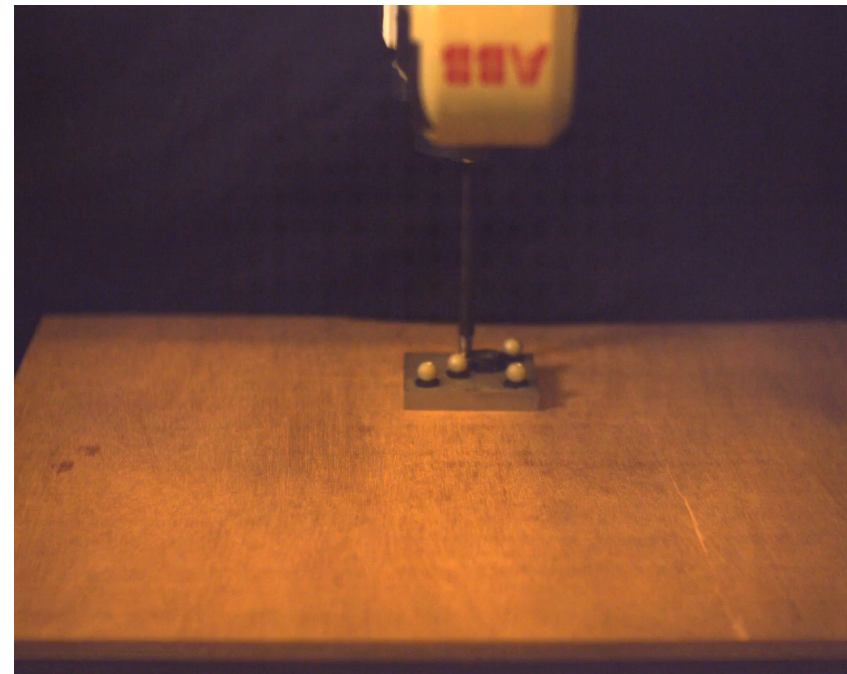
*Application to Extrinsic Dexterity*





# Summary

- ✓ Pusher-Slider **closed-loop control** with single contact.
- ✓ **Robust** to perturbations in dynamics and observations.
- ✓ Idea: Family of **primitive controllers** for manipulation.



[Hogan and Rodriguez, “*Feedback Control of the Slider-Pusher System: A Story of Hybrid and Underactuated Contact Dynamics*”, WAFR 2016]



**Massachusetts  
Institute of  
Technology**



**MCube  
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